Shear reinforcement for aseismic design of flexural members

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IS:4326-1976 provisions regarding shear design of flexural members are incomplete. This paper suggests the required specifications. The shear design must be based on plastic moment capacity of sections. A step-by-step procedure is suggested for calculation of plastic moment capacity. An alternative approximate method is also proposed. One worked out example on shear design is included.

During an earthquake, a structure should be capable of undergoing extensive inelastic deformations without a significant loss in strength. A ductile structure may yield appreciably during an earthquake. Yielding softens the structure, which increases its time period and reduces the earthquake force. Hysteretic damping also increases significantly in the inelastic range of response and this further helps to improve the earthquake response. For a structure to be ductile, none of the brittle failure modes should occur before ductile flexural failure. Shear failure is a brittle failure and must be avoided. Hence, the shear design philosophy in an earthquake-resistant structure differs significantly from that in an ordinary structure. In the latter, the members are designed for the factored moments and shears obtained from analysis for a given load combination. In the former, the design shear force will be the larger of

(*i*) shear force as obtained from analysis for given load combinations, and

(*ii*) the actual shear that is likely to develop in a member after flexural failure has taken place.

Most building codes require that the design shear force be calculated on the basis of the ultimate moment capacity of plastic hinges at the ends of the member and the factored gravity loads on the member, *Fig* 1. This ensures that, in an earthquake, brittle shear failure does not occur prior to the formation of ductile plastic hinges at the ends of the member.

The relevant clause in IS:4326-1976 (clause 7.2.5) reads, " The web reinforcement in the form of vertical stirrups shall be provided so as to develop the vertical shears resulting from all ultimate vertical loads acting on the beam plus those which

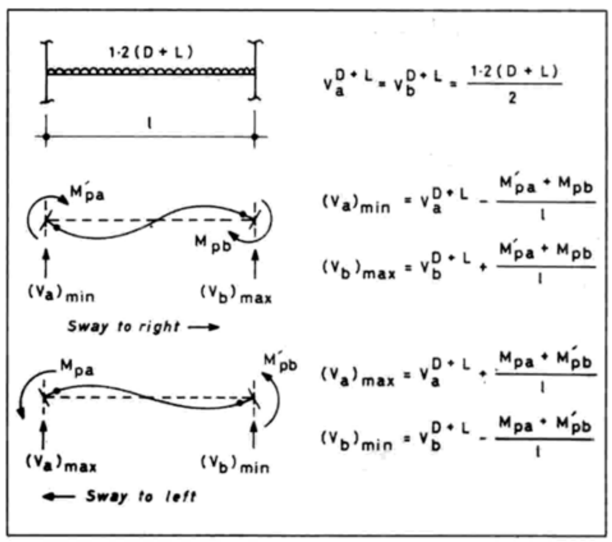

Fig 1 Calculation of design shear force

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Table 1 : Stress in the compression reinforcement, f_{sc} (N/mm^2), for a doubly reinforced section*

f_y N/mm²	$\frac{d'}{d}$			
	0.05	0.10	0.15	0.20
250	250	250	250	250
415	408	406	393	378
500	488	474	454	426

* To be used in the approximate method for computing plastic moment capacity

Table 2 : Values of plastic moment capacity obtained in design example

Moment	Mp, kNm		Percentage difference
	Exact	Approximate	
M'_{pa}	232.15	230.52	- 0.70
M_{pa}	297.31	302.88	1.88
M'_{pb}	210.35	208.56	- 0.85
M_{pb}	292.84	294.64	0.62

can be produced by the plastic moment capacities at the ends of the beam. The spacing of the stirrups shall not exceed $\frac{d}{4}$ in a length equal to $2d$ near each end of the beam and $\frac{d}{2}$ in the remaining length". However, IS:4326-1976 neither defines the term 'plastic moment capacity' nor specifies a procedure for its calculation. During a severe earthquake, the longitudinal tensile reinforcement may strain harden due to inelastic deformation of the member. This substantially increases the flexural strength of the member. Considering the above factors, it has been suggested[7] that the design shear force as per Indian codes be calculated by

(i) computing the plastic moment capacity by assuming the partial safety factor for material strength of steel and concrete as 1.0, and stress in the tension reinforcement of $1.25 f_y$, and

(ii) taking the ultimate vertical load as 1.5 times the sum of dead and live loads on the span.

However, the design shear force calculated by using the above assumptions appears to be quite conservative.

This paper suggests the alternative parameters, which are more reasonable and are analogous to ACI specifications, to be used for calculation of plastic moment capacity. Two methods are worked out to estimate the plastic moment capacity of flexural members. The first is an exact method while the second is an approximate method, appropriate for practical designs in professional work. A design example is worked out by both the methods.

Plastic moment capacity

The maximum shear force developed in a flexural member is directly proportional to the plastic moment capacity at its ends. Hence, the maximum probable plastic moment capacity, obtained due to the material strength being larger than that specified, must be used in the calculation of design shear force. The following parameters are proposed for computation of plastic moment capacity of beam sections. However, these are not to be used in calculating shear stirrup spacing for the appropriate design shear force.

(i) The partial safety factor for material strength, γ_m, may be taken as 1.0 for steel and 1.3 for concrete. The basis for these values is that SP:24, in clause 35.4.1, suggests that when considering the effects of excessive loads (for example, explosive pressures, vehicle impact), the material partial safety factors be taken as 1.3 for concrete and 1.0 for steel[10]. Moreover, ACI proposes that for calculating plastic moment capacity, strength reduction factor of 1.0 be used against 0.9 otherwise. Thus, in our case material partial safety factors of 1.3 and 1.0 against 1.5 and 1.15, respectively, are in about the same ratio as for ACI. The resulting concrete compressive stress block at ultimate condition is shown in *Fig* 2.

(ii) The tensile stress in the longitudinal reinforcement may be taken as $1.25 f_y$ on yielding. This is due to the likelihood of actual yield stress of steel being greater than the specified nominal yield stress and the possibility of strain hardening in the tension steel when plastic hinges appear at the beam ends. This results in an increase in the plastic moment capacity and hence the maximum shear force. This is consistent with the ACI code which also recommends $1.25 f_y$ as the steel stress for calculation of plastic moment capacity. The stress in steel for a given value of strain is to be read from the characteristic stress-strain curve up to the yield strain. Beyond yield strain, stress in tension and compression steel is taken as 1.25 f_y and f_y, respectively. The yield strain, ϵ_y, is taken as 0.00125 for mild steel (Fe 250) and 0.0038

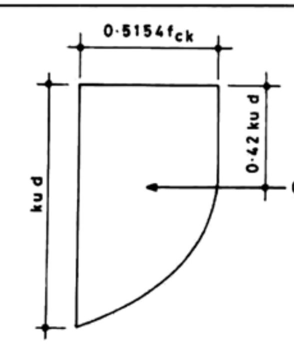

Fig 2 Stress block parameters for computation of plastic moment capacity

for high-yield strength deformed (HYSD) bars (Fe 415).

It is also proposed that the factored vertical load for calculation of design shear force be taken as 1.2 times the sum of dead and live loads on the span.

Analysis

Singly reinforced section

Consider a singly reinforced section loaded to its flexural strength (*Fig* 3). Let

b = breadth of section

d = effective depth of section

f_{ck} = characteristic strength of concrete

Ku = non-dimensional depth of the neutral axis

A_{st} = area of tension reinforcement

ϵ_{cu} = ultimate crushing strain in concrete (= 0.0035)

f_y = yield stress of steel and

E_s = elastic modulus of steel.

The compressive force, C, and tensile force, T, are given by

$$C = 0.4172\ f_{ck}\ Ku\ b\ d \tag{1}$$

$$T = 1.25 f_y\ A_{st} \tag{2}$$

Equating C with T for equilibrium of the section

$$Ku = \left[\frac{1.25\ f_y\ P_t}{0.4172\ f_{ck}} \right] \tag{3}$$

where P_t is the ratio of tension steel (A_{st}/bd).

In earthquake resistant design, over-reinforced sections being brittle are to be always avoided. Hence, presuming the section to be under-reinforced, the plastic moment capacity is given by

$$\frac{M_p}{bd^2} = 1.25\ f_y\ P_t\ (1.0 - 0.416\ Ku) \tag{4}$$

where K_u is to be taken from equation (3).

Doubly reinforced section

Consider a doubly reinforced section loaded to its flexural strength (*Fig* 4). Let

A_{sc} = area of compression reinforcement

P_c = ratio of compression steel (A_{sc}/bd)

ϵ_{sc} = strain in the compression steel

f_{sc} = stress in the compression steel, and

d' = effective cover to compression steel.

The analysis differs depending on whether one or both of the tensile and compressive reinforcements yield.

Case I : Tension steel yields

Initially assume that the tension steel yields. This assumption is checked later. Further, assume that the compression steel is in the elastic range. Expressions for C and T are obtained as:

$$C = 0.4172\ f_{ck}\ Ku\ b\ d + 0.0035\ E_s \left[1.0 - \frac{d'}{d}\frac{1}{Ku} \right] A_{sc} \tag{5}$$

$$T = f_y\ A_{st} \tag{6}$$

Equating these for equilibrium, the following quadratic equation is obtained for Ku

$$\alpha\ Ku^2 + \beta\ Ku - \delta = 0 \tag{7}$$

where $\alpha = 0.4172\ f_{ck}$; $\beta = 700\ P_c - f_y\ P_t$; and $\delta = 700\ \frac{d'}{d}\ P_c$. The value of Ku from this equation is used to find strain in the tension steel, which is given by

$$\epsilon_{st} = 0.0035 \left[\frac{1}{Ku} - 1.0 \right] \tag{8}$$

If ϵ_{st} is found to be greater than ϵ_y, then the tension steel has indeed yielded and further calculations are done assuming stress in the tension reinforcement as $1.25\ f_y$. Equilibrium equation, $C = T$, with C from equation (5) and $T = 1.25\ f_y\ A_{st}$, gives another quadratic equation

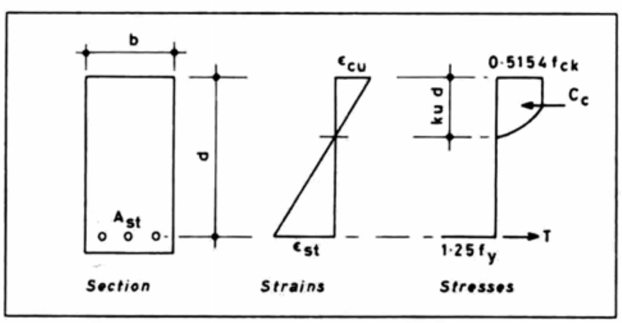

Fig 3 Singly reinforced section at its flexural strength

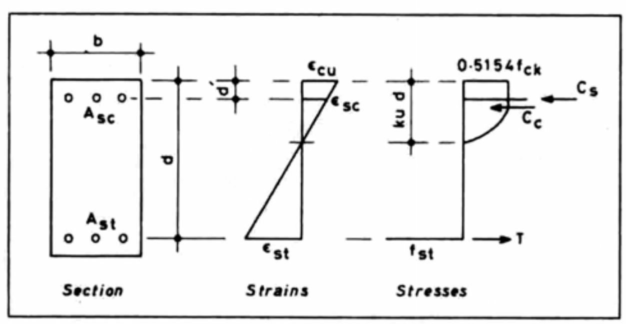

Fig 4 Doubly reinforced section at its flexural strength

$$\alpha \ Ku^2 + \beta \ Ku - \delta = 0 \quad (9)$$

where $\alpha = 0.4172 \ f_{ck}$; $\beta = 700 \ P_c - 1.25 \ f_y \ P_t$; and $\delta = 700 \frac{d'}{d} P_c$. Use this value of Ku to calculate strain in the compression steel, which is given by

$$\epsilon_{sc} = 0.0035 \left[1.0 - \frac{d'}{d} \frac{1}{Ku} \right] \quad (10)$$

If ϵ_{sc} is less than ϵ_y, then take f_{sc} from the stress strain curve for the calculated value of ϵ_{sc}. Otherwise, i.e. if ϵ_{sc} is greater than ϵ_y, the compression steel has also yielded and f_{sc} is to be taken as f_y. Note that the factor of 1.25 is not used here to increase f_y because the compression steel will not strain harden at the strain of about 0.0035 at which concrete crushes. Now, using the appropriate value of f_{sc}, calculate the value of Ku from the following equilibrium equation

$$Ku = \left[\frac{1.25 \ f_y \ P_t - f_{sc} \ P_c}{0.4172 \ f_{ck}} \right] \quad (11)$$

The plastic moment capacity of the section is now obtained as

$$\frac{M_p}{bd^2} = 0.4172 \ f_{ck} \ Ku \ (1.0 - 0.416 \ Ku) + f_{sc} \ P_c \left[1.0 - \frac{d'}{d} \right] \quad (12)$$

Case II : Tension steel does not yield

In case ϵ_{st}, calculated by equation (8), indicates that the tension steel does not yield, an iterative approach is required. The position of the neutral axis, Ku, is initially assumed at the level of the compression steel. This value of Ku is substituted in equations (8) and (10) to obtain the strains, ϵ_{st} and ϵ_{sc}, respectively. The corresponding stresses, f_{st} and f_{sc}, are found. The tensile force, T, and the compressive force, C, are given by

$$T = f_{st} \ A_{st}$$

$$C = 0.4172 \ f_{ck} \ Ku \ b \ d + f_{sc} \ A_{sc} \quad (13)$$

If T is greater than C, the value of Ku is given a small increment and the procedure is repeated. A stage is reached when C and T become almost equal. This value of Ku is used to obtain the plastic moment capacity from equation (12).

Approximate method of analysis

The formulation discussed above is tedious for design office application to a doubly reinforced section as it requires the solution of quadratic equations and may become an iterative process when the tension steel does not yield. Hence, approximate methods are used to estimate the plastic moment capacity of flexural members. References 3 and 4 neglect the compression steel while calculating the plastic moment capacity of doubly reinforced sections. The plastic moment capacity thus obtained is then increased by 5 percent to account for the compression steel. This procedure has been compared with the aforementioned exact analysis for various sections and it was found to give reasonable results only for sections having a low percentage of tension steel. Therefore, an alternative approximate method for the analysis of a doubly reinforced section has been developed and is presented here.

Let P_{tb} be the tension steel ratio (A_{st}/bd) for a singly reinforced balanced section as per provisions of IS:456-1978. Similarly, let P_{cb} be the compression steel ratio (A_{sc}/bd) for a doubly reinforced balanced section as per IS:456-1978 for a tension steel ratio of P_t. If the tension steel ratio of the section, P_t, is less than P_{tb}, the section is under-reinforced, irrespective of the compression steel ratio present. For a section that has tension steel ratio, P_t, greater than P_{tb}, if the compression steel ratio, P_c, is greater than P_{cb}, the section is under-reinforced; otherwise it is over-reinforced. Let the total tension steel ratio, P_t, consists of two parts, P_{t1} and P_{t2}. Here, P_{t2} is such as to balance the force due to the compression steel only and P_{t1} is equal to the difference of P_t and P_{t2}. Further, let the plastic moment capacity, M_p, be comprised of two parts M_{p1} (due to concrete and P_{t1}) and M_{p2} (due to P_c and P_{t2}). The following two cases arise depending on whether the section is under-reinforced or over-reinforced.

Case I : Under-reinforced section

(i) Obtain f_{sc} from *Table* 1. Stress values in *Table* 1

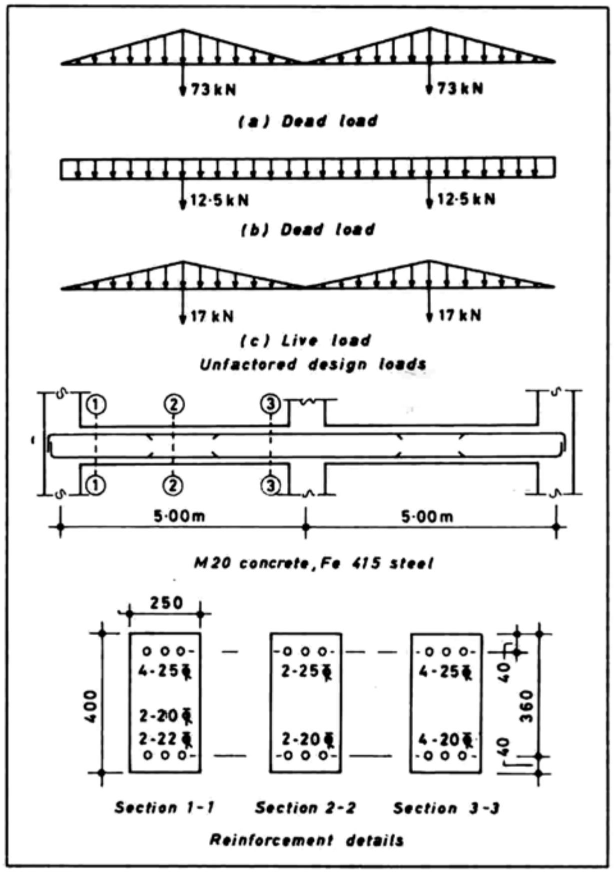

Fig 5 Design example

have been obtained by multiplying stress values of *Table F* of SP:16[8] by 1.15 to make the partial safety factor for material strength of compression steel as 1.0 against 0.87.

(ii) Find $P_{t2} = \left[\frac{f_{sc}\ P_c}{1.25\ f_y} \right]$ (14)

If P_{t2} is greater than P_t, set $P_{t2} = P_t$.

(iii) Calculate $P_{t1} = P_t - P_{t2}$

(iv) Obtain M_{p1} from equation (4) by substituting P_{t1} as P_t.

(v) Obtain M_{p2} from the following

$$\frac{M_{p2}}{bd^2} = 1.25\ f_y\ P_{t2} \left[1.0 - \frac{d'}{d} \right] \quad (15)$$

(v) $M_p = M_{p1} + M_{p2}$ (16)

Case II : Over reinforced section

(i) Obtain M_{p1} from equation (4) by substituting P_{tb} as P_t

(ii) Obtain M_{p2} from the following

$$\frac{M_{p2}}{bd^2} = f_{sc}\ P_c \left[1.0 - \frac{d'}{d} \right] \quad (17)$$

where f_{sc} is taken from *Table* 1.

(iii) $M_p = M_{p1} + M_{p2}$

An extensive parametric study was conducted to evaluate the accuracy of the proposed approximate method. The plastic moment capacity was computed by the exact formulation as well as the approximate method for sections having area of compression and tension steel varying from minimum permissible as per IS:456-1978 to a maximum of 0.04 *bd*. It is observed that for sections which have tension steel ratio less than P_{tb}, the approximate method underestimates the plastic moment capacity by not more than 5 percent, irrespective of the ratio of compression steel present. For doubly reinforced sections which are under-reinforced as per IS:456-1978, the approximate method gives results within 2.5 percent of the exact method. For doubly reinforced sections that are over-reinforced as per IS:456-1978, the approximate method may underestimate the plastic moment capacity by a maximum of 17 percent. However, such over-reinforced sections are not to be used, particularly in aseismic design. Thus the proposed approximate method is reasonably accurate and is quite simple to use.

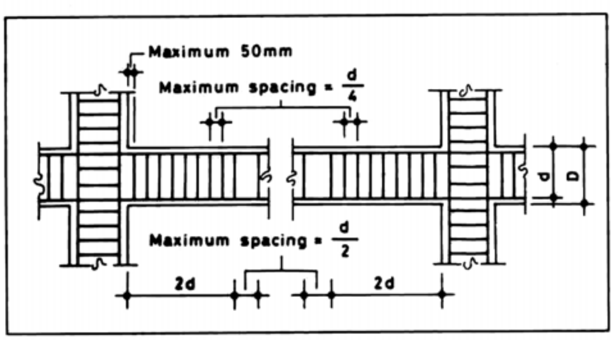

Fig 6 Provision of shear reinforcement

Example

Consider a beam shown in *Fig* 5 which is part of a moment resisting frame of a 5-storeyed building. In the following, the shear reinforcement of the beam has been designed for illustration.

Plastic moment capacity (Exact method)

Calculation of M'_{ps}: $\frac{d'}{d} = 0.11$, $P_c = 0.0218$ and $P_t = 0.0154$.

Using equation (7), $Ku = 0.1655$. Equation (8) gives $\epsilon_{st} = 0.0176$, which is greater than ϵ_y (0.0038). Therefore the tension steel yields. Equation (9) gives $Ku = 0.1914$. From equation (10), $\epsilon_{sc} = 0.0015$, which is less than ϵ_y. Hence, $f_{sc} = \epsilon_{sc}\ E_s = 293.7$ N/mm^2. From equation (11), $Ku = 0.1913$. Using equation (12), $M_p/bd^2 = 7.1652$ and $M'_{ps} = 232.15$ kNm. The values of M_{ps}, M'_{pb} and M_{pb} are similarly calculated. These values are given in *Table* 2.

Plastic moment capacity (Approximate method)

Calculation of M'_{ps}: The balanced tension steel ratio (P_{tb}) as per IS:456-1978 for a singly reinforced section for M20 and Fe 415 combination is 0.0096. A compression steel ratio of 0.0060 (P_{cb}) is required in order to obtain a balanced, doubly reinforced section as per IS:456-1978, with the actual tension steel ratio (P_t) of 0.0154. As P_{cb} is less than the available compression steel ratio, P_c (= 0.0218), the section is under-reinforced. From equation (14), $P_{t2} = 0.0171$. As P_{t2} is greater than the available tension steel ratio P_t (= 0.0154), take $P_{t2} = P_t$. Hence $P_{t1} = P_t - P_{t2} = 0$. From equation (4), $M_{p1}/bd^2 = 0.0$ and from equation (15), $M_{p2}/bd^2 = 7.1149$. From equation (16), $M_p/bd^2 = 7.1149$ and $M'_{ps} = 230.52$ kNm. The values of M_{ps}, M'_{pb} and M_{pb} are calculated in a similar manner and are given in *Table* 2. These values are quite close to those by the exact method.

Design shear force

Using a load factor of 1.2 for dead and live loads, the factored load on the span is 123 kN. The shear due to the earthquake force will depend on the direction of sway (*Fig* 1).

The following two cases arise.

Case I : Sway to the right

$V_a^{D+L} = V_b^{D+L} = 61.5$ kN; $(M'_{ps} + M_{pb})/l = 105$ kN. Hence, $(V_a)_{min} = 61.5 - 105 = -43.5$ kN and $(V_b)_{max} = 61.5 + 105 = 166.5$ kN.

Case II : Sway to the left

$(M_{ps} + M'_{pb})/l = 101.53$ kN. Hence, $(V_a)_{max} = 61.5 + 101.53 = 163.03$ kN and $(V_b)_{min} = 61.5 - 101.53 = -40.03$ kN.

Thus, the design shear forces are $(V_a)_{max} = 61.5 + 101.53 = 163.03$ kN and $(V_b)_{min} = 61.5 - 101.53 = -40.03$ kN.

Thus, the design shear forces are $(V_a)_{max} = 163.03$ kN and $(V_b)_{max} = 166.5$ kN at ends A and B, respectively. The maximum negative shear forces are $(V_a)_{min} = -43.5$ kN and $(V_b)_{min} = -40.03$ kN at ends A and B, respectively. It is evident that the seismic shear force can reverse direction. Hence, use of vertical stirrups and careful detailing at the beam ends are essential.

Stirrup spacing

The design shear force is 166.5 kN. From *Table* 14, IS:456-1978, the maximum shear that the section can carry with suitable shear reinforcement is 252.0 kN. From *Table* 13 of IS:456-1978, the shear that can be carried by the concrete alone with 1.4 percent tension reinforcement (section 3-3) is 63.0 kN. The shear to be carried by the vertical reinforcement, V_s, is 166.5 - 63.0 = 103.5 kN. Using 8-mm diameter two legged stirrups the spacing is 125mm. As per IS:4326-1976, the stirrup spacing should not exceed $\frac{d}{4}$ (= 90mm) in a length equal to $2d$ near each end of the beam and $\frac{d}{2}$ (= 180mm) elsewhere (*Fig* 6). Hence, provide vertical stirrups at a spacing of 90mm in a length of 750mm at either end of the beam and at a spacing of 180mm for the remaining length. American practice is to provide the first stirrup at not more than 50mm from the column face and it is recommended that this be followed in India also.

Summary and conclusions

The design shear force for flexural members in an earthquake resistant structure is to be found by considering the ultimate moment capacity of plastic hinges at the ends of the member and ultimate vertical load on the span. IS:4326-1976 does not specify the procedure to calculate the plastic moment capacity. This paper recommends use of partial safety factor for material strength of 1.0 and 1.3 for steel and concrete, respectively, and stress in the tension steel of 1.25 f_y to compute the plastic moment capacity. The ultimate vertical load for calculation of design shear force may be taken as 1.2 times the sum of dead and live loads on the span. Using these parameters, a step by step procedure is outlined to calculate plastic moment capacity. However, for convenience in design offices, an alternative approximate method is also presented. The proposed approximate method is quite accurate for under-reinforced sections but may considerably underestimate the plastic moment capacity of over-reinforced sections. However, over-reinforced sections are not to be used in aseismic design of structures due to poor ductility. An example is shown for design of shear reinforcement as per ductility requirement.

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